Superconducting film with randomly magnetized dots: A realization of the two-dimensional *XY* model with random phase shifts

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We consider a thin superconducting film with randomly magnetized dots on top of it. The dots produce a disordered pinning potential for vortices in the film. We show that for dots with permanent and random magnetization normal or parallel to the film surface, our system is an experimental realization of the two-dimensional *XY* model with random phase shifts. The low-temperature superconducting phase, that exists without magnetic dots, survives in the presence of magnetic dots for sufficiently small disorder.

I. INTRODUCTION

Since the pioneering papers by Berezinskii[1] and Kosterlitz and Thouless[2] finite-temperature phase transitions in two-dimensional (2D) systems, which have a continuous symmetry specified by a phase, have become a very active research field. It turned out that two-dimensional superfluids[3] as well as thin superconducting films[4–7] have a Berezinskii-Kosterlitz-Thouless (BKT) transition at a finite temperature T_{2D}. The low-temperature phase is superfluid (superconducting) and has quasilong-range order while the disordered high temperature phase is normal liquid (metallic) for superfluids (superconductors). These systems are successfully described by the 2D *XY* model, see Eq. (1) with $A_{ij} \equiv 0$. Theoretical predictions were confirmed experimentally: a universal jump in the superfluid density at T_{2D} (Ref. 8) and different current-voltage characteristics of superconducting films below and above T_{2D}.[9]

Introducing disorder through random phase shifts in the 2D *XY* model physics becomes more complex. The Hamiltonian of the 2D *XY* model with random phase shifts reads[10,11]

$$\mathcal{H} = -\frac{\varepsilon_0}{\pi}\sum_{i,j}\cos(\phi_i - \phi_j - A_{ij}), \tag{1}$$

where the sum runs over all nearest-neighboring sites on a square lattice. ϕ_i denotes the phase of the order parameter while A_{ij} are quenched random phase shifts on bonds of the square lattice produced by some kind of disorder. We assume that A_{ij} are Gaussian distributed, uncorrelated, have the zero mean $\langle A_{ij}\rangle = 0$ and the variance $\langle A_{ij}^2\rangle = \sigma$. The pure 2D *XY* model contains two different kinds of excitations which are decoupled. The spin-wave excitations describe small changes in the phase ϕ and do not drive any phase transitions. Another kind of excitations are vortices, topological excitations, and they are essential for the existence of a BKT transition. Vortices are bound in pairs in the low-temperature phase and unbound at high temperatures. Introducing the disorder in the form of random phase shifts, the low-temperature superconducting phase survives for weak enough disorder, as has been shown first analytically[11–14] and then numerically.[15,16] The phase diagram of the model (1) is given in Fig. 1.

Hamiltonian (1) describes the thermodynamic behavior of several disordered systems, including two-dimensional ferromagnets with random Dzyaloshinskii-Moriya interaction,[10] Josephson-junction arrays with positional disorder[20] and vortex glasses.[21] In this paper we will consider another system and show that it also belongs to the class of systems that can be described by the model (1). It is a ferromagnet-superconductor hybrid system. The system consists of a thin superconducting film covered by magnetic dots with permanent but random magnetization. For more details about hybrid systems see recent reviews.[22,23] We will show that our system can be described by Hamiltonian (1) under some conditions defined below. Knowing the solution of the model (1) we can establish possible phases of the system.

Placed on top of the film, a single magnetic dot with sufficiently large magnetization, normal to the film surface, induces and pins vortices and antivortices around it.[24–27] A regular lattice of magnetic dots with constant and equal magnetization is the source of a periodic pinning potential for vortices in the film. The magnetic dots induce periodic arrays of vortices and antivortices in the film, provided the dot magnetization is high enough.[28,29] The periodic pinning is absent in the case of a random-dot magnetization, and there magnetic dots are a source of disordered pinning potential for vortices. Lyuksyutov and Pokrovsky[22,24] considered a superconducting film with a magnetic dots array with random, sufficiently strong magnetic moments and concluded that the dot array induces the resistive state in the film. They have not considered the case when magnetic moments are weak. In this paper we will show that the superconducting state survives provided the disorder is not too strong. Hybrid systems with a regular lattice of magnetic dots with random magnetization,[30] and without a lattice, but homogeneously distributed dots on top of a thin superconducting film[31] have been recently experimentally realized.

In the rest of the paper in Sec. II we introduce the model for our hybrid system and give its solution by mapping it to the model (1). Section III contains discussions and conclusions. Some technical details are postponed to the Appendix.

II. MODEL AND ITS SOLUTION

We consider a thin superconducting film characterized by the London penetration depth λ_L, the coherence length ξ, a
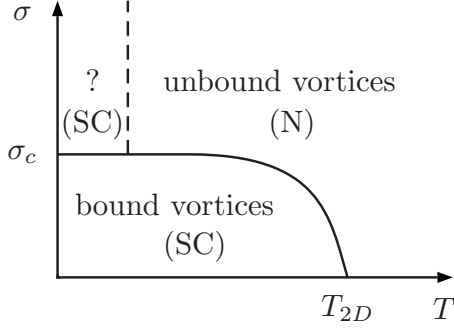
FIG. 1. Phase diagram of the 2D *XY* model with random phase shifts. At low temperature and weak disorder $\sigma < \sigma_c$ a superconducting phase occurs, and there vortices are bound in pairs. Above T_{2D} vortices are unbound and the phase is nonsuperconducting (N). According to some studies (Refs. [17]–[19]) at low temperatures and for strong enough disorder appears another superconducting phase.

thickness d, and a typical lateral dimension L. The London penetration depth is in general temperature dependent and we assume that our film has the effective penetration depth $\lambda = \lambda_L^2/d$ that exceeds film's lateral dimension L. This limit is valid for "dirty" superconducting films.[4] In addition, provided the bulk critical temperature is larger than a critical temperature T_{2D} for vortex unbinding, the film has a BKT transition at T_{2D}.[4,5]

We consider the dots with permanent random magnetization placed on top of the film. They produce a random potential V for vortices in the film. Assuming vortices of vorticities n_i are places on a quadratic lattice with the lattice constant a (which is of the order of the coherence length), the effective lattice Hamiltonian for the system may be written as

$$\mathcal{H}_v = \sum_i \left[n_i^2 (E_c + U_v) + n_i V_i \right] + \frac{1}{2} \sum_{i \neq j} n_i n_j U_{vv}(\rho_{ij}), \quad (2)$$

where the sum runs over all lattice sites, $\varepsilon_0 = \phi_0^2/(16\pi^2\lambda)$, the flux quantum is $\phi_0 = hc/(2e)$ and V_i is the random potential at site i while E_c is the single-vortex core energy which is of the order ε_0. U_v and $U_{vv}(\rho_{ij})$ are the single-vortex energy and the interaction energy of two vortices separated by a distance ρ_{ij}, respectively, and for $L < \lambda$ read[32,33]

$$U_v = \varepsilon_0 \ln\frac{L}{a}, \quad U_{vv}(\rho_{ij}) = 2\varepsilon_0 \ln\frac{L}{\rho_{ij}}. \quad (3)$$

Using the expressions in Eq. (3) it is useful to rewrite the Hamiltonian (2) in the form

$$\mathcal{H}_v = \sum_i (n_i^2 E_c + n_i V_i) - \varepsilon_0 \sum_{i \neq j} n_i n_j \ln\frac{\rho_{ij}}{a} + N^2 U_v, \quad (4)$$

where we have introduced the total vorticity of the system $N = \Sigma_i n_i$. In the limit $L \gg a$ and without the disorder potential $V_i = 0$, the last term in Eq. (4) penalizes the total energy for nonzero total vorticities[34,35] so one has $N = 0$. Then, a superconducting film described by the model (4) has a BKT transition at the temperature $T_{2D} = \varepsilon_0(T_{2D})/2$, where $\varepsilon_0(T_{2D})$ de-

notes that one should take value $\lambda(T_{2D})$ renormalized by the presence of vortices.[2,4,5]

Next, we consider effects of the random potential. First we consider the case when the dots have random magnetization parallel to the film surface. We model them as magnetic dipoles placed on top of the film at lattice sites. To characterize statistical properties of the dots, we assume that the x and y components of the magnetic moment at site i are Gaussian distributed, have zero mean value and are uncorrelated from site to site,

$$\langle m_{i\alpha} \rangle = 0, \quad \langle m_{i\alpha} m_{j\beta} \rangle = \mathcal{M}^2 \delta_{ij} \delta_{\alpha\beta}, \quad \alpha, \beta = x, y, \quad (5)$$

where $\langle \cdots \rangle$ denotes an average over disorder. Since the dots have random magnetic moments, $\mathcal{M}$ is the measure for a typical magnetic moment of a magnetic dot at some lattice site.

The interaction energy between a single dot having the magnetic moment **m** parallel to the film surface and a vortex of vorticity n placed at a relative distance $\boldsymbol{\rho}$ from the dot can be calculated using the approach developed in Ref. [36], and reads[27]

$$U_{mv}^\parallel(n,\rho) = \frac{\mathbf{m} \cdot \boldsymbol{\rho}}{\rho} \frac{n\phi_0}{2\pi} \int_0^\infty dk \frac{kJ_1(k\rho)}{1 + 2\lambda k}$$

$$= \frac{\mathbf{m} \cdot \boldsymbol{\rho}}{\rho} \frac{n\phi_0}{16\lambda^2} \left[H_1\left(\frac{\rho}{2\lambda}\right) - Y_1\left(\frac{\rho}{2\lambda}\right) - \frac{2}{\pi} \right], \quad (6)$$

where J_1 is the Bessel function of the first kind, H_1 is the Struve function, and Y_1 is the Bessel function of the second kind.[37] Notice that the interaction energy between the magnetic dipole and the vortex can be simply written in the form $-\mathbf{m} \cdot \mathbf{B}_v$, where $\mathbf{B}_v$ is the magnetic field produced by the vortex at the dipole position.[27,38] In the limit $\rho \ll \lambda$ the interaction energy [Eq. (6)] reads

$$U_{mv}^\parallel(n,\rho) = \frac{\mathbf{m} \cdot \boldsymbol{\rho}}{\rho} \frac{n\phi_0}{4\pi\lambda\rho}. \quad (7)$$

The random site potential V_i is given as a sum over the lattice

$$V_i = \sum_{j \neq i} \frac{U_{mv}^\parallel(n_j, \rho_{ij})}{n_j} = \frac{\phi_0}{4\pi\lambda} \sum_{j \neq i} \frac{\mathbf{m}_j \cdot \boldsymbol{\rho}_{ij}}{\rho_{ij}^2}. \quad (8)$$

By summing over $j \neq i$ in the previous expression we avoid the short-scale cutoff divergence of the interaction energy [Eq. (6)] at $\rho = 0$, which exists because the dots are placed at the top of the film in our model. In reality magnetic dipoles are separated from the film surface by some small distance $\sim a$. Since V_i is a sum of many independent random variables it is Gaussian distributed (notice here that the assumption about Gaussian distribution for $m_{i\alpha}$ is not necessary condition for V_i to be Gaussian distributed; it is sufficient that dots have a distribution with a finite variance). Its mean, variance, and site to site correlations can be calculated and read as

$$\langle V_i \rangle = 0, \quad (9)$$

$$\langle V_i^2 \rangle = 2\pi \frac{\mathcal{M}^2 \varepsilon_0}{\lambda a^2} \ln\frac{L}{a} + \mathcal{O}(1), \quad (10)$$

$$\langle (V_i - V_j)^2 \rangle = 4\pi \frac{\mathcal{M}^2 \varepsilon_0}{\lambda a^2} \ln \frac{\rho_{ij}}{a} + \mathcal{O}(1). \qquad (11)$$

The model (4) with the disorder potential (8) which has properties given by Eqs. (9)–(11) matches the vortex part of the 2D *XY* model with random phase shifts.[10,39] From the solution of the model (1) (Refs. 11, 14, and 39) we know its phase diagram, see Fig. 1. At zero temperature there is a critical value for the typical magnetic moment per unit length,

$$\frac{\mathcal{M}_c}{a} = \frac{\phi_0}{8\pi\sqrt{2\pi}}. \qquad (12)$$

For $\mathcal{M} < \mathcal{M}_c$ the system has no free vortices $N=0$ while for $\mathcal{M} > \mathcal{M}_c$ the disorder spontaneously creates and induces unbound vortices, and one generally has $N \neq 0$. A simple argument[40] based on Eq. (4) which compares the energy loss for the single-vortex creation and the energy gain due to disorder fluctuations also gives the critical value [Eq. (12)] for zero temperature. The connection between σ introduced as the disorder strength in Eq. (1) and the typical magnetic moment $\mathcal{M}$ introduced in Eq. (5) is $\sigma = (\frac{4\pi^2 \mathcal{M}}{\phi_0 a})^2$ while the random phase is $\mathbf{A}_i = \frac{4\pi^2}{\phi_0 a} \mathbf{e}_z \times \mathbf{m}_i$, where the x (y) component of $\mathbf{A}_i$ corresponds to the disorder A_{ij} on horizontal (vertical) bond at site i.

In the case that the dot magnetization is normal to the film surface, similar to Eq. (6), for the interaction energy between a dot of magnetization m and a vortex of vorticity n separated by a distance ρ we get

$$U_{mv}^{\perp}(n,\rho) = m \frac{n\phi_0}{2\pi} \int_0^{\infty} dk \frac{kJ_0(k\rho)}{1+2\lambda k}$$

$$= m \frac{n\phi_0}{16\lambda^2} \left[Y_0\left(\frac{\rho}{2\lambda}\right) - H_0\left(\frac{\rho}{2\lambda}\right) - \frac{4\lambda}{\pi\rho} \right]. \quad (13)$$

The previous expression simplifies for $\rho \ll \lambda$ and reads

$$U_{mv}^{\perp}(n,\rho) = m \frac{n\phi_0}{4\pi\lambda\rho}. \qquad (14)$$

By assuming that the film is covered by magnetic dots with random magnetization normal to the film surface (along the $\hat{\mathbf{z}}$ direction), satisfying

$$\langle m_{iz} \rangle = 0, \quad \langle m_{iz} m_{jz} \rangle = \mathcal{M}^2 \delta_{ij}, \qquad (15)$$

for the site random potential we get

$$V_i' = \sum_{j \neq i} \frac{U_{mv}^{\perp}(n_j, \rho_{ij})}{n_j} = \frac{\phi_0}{4\pi\lambda} \sum_{j \neq i} \frac{m_{jz}}{\rho_{ij}}, \qquad (16)$$

which, as we show in the Appendix, is equivalent to V_i and hence has statistical properties given by Eqs. (9)–(11). We may conclude that the system with magnetic moments normal to the film has the same phase diagram as in the case of moments parallel to the film surface.

III. DISCUSSIONS AND CONCLUSIONS

Having shown the equivalence between our system and the vortex part of the 2D *XY* model with random phase

shifts, we may infer some properties of the former. The phase diagram is given in Fig. 1. The low-temperature and low disorder phase is superconducting. There vortices and antivortices are bound in pairs. The current-voltage characteristic for $T \to T_{2D}$ and $T < T_{2D}$ and for weak disorder is expected to be very similar to the one of Halperin and Nelson[7] for the pure case, $V \sim I^3$, possibly with a small correction due to the disorder. This phase has zero linear resistivity. The high-temperature phase is metallic and has a nonzero linear resistivity. There free vortices dissipate energy and produce the linear current-voltage characteristic $V \sim I$.

In Ref. 24 the conclusion about the resistive state of the film when the dots with normal magnetization are present relies on the assumption that the randomly magnetized dots pin vortices of vorticity ± 1. These pinned vortices serve as a source of the random potential for other bound vortices, which unbind and fill deep valleys of the random potential. These unbound vortices lead to the resistive state of the film. We agree that this scenario occurs for sufficiently strong disorder when the dots can induce and pin vortices. A single dot can induce and pin quite different configurations of vortices and antivortices regarding its magnetic moment.[27] We expect that a random lattice of dots can also pin, from site to site, quite a different number of vortices and antivortices which produce different potential than one assumed in Ref. 24. However, our conclusion, that the resistive state in films occurs when the disorder is sufficiently strong, agrees with the one from Ref. 24. Moreover, we give the strength of the disorder above which the resistive state occurs.

By making a comparison between $\mathcal{M}_c$ and the magnetic moment m_{1c} of a single magnetic dot with normal magnetization necessary to induce and pin an extra vortex in the film, we obtain $m_{1c} \approx \mathcal{M}_c \sqrt{8\pi} (b/a) \ln(L/a)$, where b is the distance between the dipole and the film surface. In addition, knowing that the value $\mathcal{M}_c$ corresponds not to typical but rare magnetic moments from the tail of distribution,[11,40] we may conclude that even a very rare magnetic dot in the film that has the magnetic moment $\mathcal{M}_c$ is not able to induce and pin vortices. Such pinned vortices served as a source of random potential in Ref. 24.

In this paper we have considered the dots as magnetic dipoles. This fact is unimportant as long as the dot size is not too big with respect to the lattice constant. What is crucial for any kind of magnetic dots is their interaction with a vortex which decays as $1/\rho$, which is universal for any geometrical shape of dots, when the vortex is sufficiently far from the dot. The magnetic field produced by a vortex decays as $1/\rho$ and the interaction-energy dot vortex universally decays, regardless of the shape of the dot. This form of the interaction produces logarithmically diverging, with the system size, variance of the disorder potential that is characteristic for Hamiltonian (1).

Recently, the question of a possible third phase for strong disorder and at low temperatures has been raised in numerical studies of the model (1) with uniformly random phase A_{ij} in the interval $[-r\pi, r\pi]$ with $0 \leq r \leq 1$, with conflicting results about its existence.[17,19,41] While for $r < r_c \approx 0.37$ (Ref. 16) it is accepted that the superconducting phase survives at low temperature, the case $r > r_c$ is still under debate. An experimental study of Josephson-junction arrays with posi-

tional disorder[18] supports the existence of the third phase. The new phase is superconducting according to the experiments of Ref. 18 and numerical investigations of Ref. 17. In the limit $\sigma \rightarrow \infty$ the model (1) and the so-called gauge glass limit, $r=1$, are equivalent, and we expect that the conjectures about the phase diagram for the case of uniformly distributed phase apply as well to the Gaussian distribution of the phase. The existence of the third phase is a challenging question that could be experimentally resolved by using magnetosuperconducting hybrid systems as a realization for the model (1).

To conclude, we have shown that a thin superconducting film covered by magnetic dots with random magnetization provides an experimental realization for the two-dimensional *XY* model with random phase shifts. The phase diagram of the latter model helped us to conclude that a low-temperature superconducting phase of a superconducting film without dots survives when the dots are placed on top of the film, provided their magnetization is not too large.

APPENDIX

In this Appendix we prove that expressions for the disorder potential produced by magnetic moments parallel to the film surface [Eq. (8)] and normal to the film surface [Eq. (16)] are equivalent. Rewriting the expression $\mathbf{m}_j \cdot \boldsymbol{\rho}_{ij}/\rho_{ij}^2$ from Eq. (8) as $m_j \cos(\alpha_j - \alpha_\rho)/\rho_{ij}$, where $\cos \alpha_j = \mathbf{m}_j \cdot \mathbf{e}_x/m_j$ and $\cos \alpha_\rho = \boldsymbol{\rho}_{ij} \cdot \mathbf{e}_x/\rho_{ij}$, we will in the following show that the distribution of random variable $m_{rj} = m_j \cos(\alpha_j - \alpha_\rho)$ (which is the projection of $\mathbf{m}_j$ onto $\boldsymbol{\rho}_{ij}$) is Gaussian, with zero mean and the variance $\mathcal{M}^2$.

By assumption (5), the components of the magnetic moment m_{jx} and m_{jy} are Gaussian distributed and have the distribution function,

$$p(t) = \frac{1}{\sqrt{2\pi}\mathcal{M}} \exp\left(-\frac{t^2}{2\mathcal{M}^2}\right), \quad t = m_{jx}, m_{jy}. \quad \text{(A1)}$$

Then the distribution function of the magnetic moment $m_j = \sqrt{m_{jx}^2 + m_{jy}^2}$ is

$$p(m_j) = \int_{-\infty}^{\infty} dm_{jx} \int_{-\infty}^{\infty} dm_{jy} p(m_{jx}) p(m_{jy})$$

$$\times \delta(m_j - \sqrt{m_{jx}^2 + m_{jy}^2}) = \frac{m_j}{\mathcal{M}^2} \exp\left(-\frac{m_j^2}{2\mathcal{M}^2}\right) \quad \text{(A2)}$$

while the angle α_j between m_{jx} and m_j is uniformly distributed in the interval $[0, 2\pi)$ and has the distribution $p(\alpha_j) = 1/(2\pi)$. The distribution of the random variable m_{rj} is

$$p(m_{rj}) = \int_0^{\infty} dm_j \int_0^{2\pi} d\alpha_j p(m_j) p(\alpha_j)$$

$$\times \delta[m_{rj} - m_j \cos(\alpha_j - \alpha_\rho)]. \quad \text{(A3)}$$

The previous expression can be most easily evaluated first by taking the Fourier transform of $p(m_{rj})$,

$$\tilde{p}_k = \int_0^{\infty} dm_j \int_0^{2\pi} d\alpha_j \frac{m_j}{2\pi\mathcal{M}^2} \exp\left(-\frac{m_j^2}{2\mathcal{M}^2}\right)$$

$$\times \exp[ikm_j \cos(\alpha_j - \alpha_\rho)]$$

$$= \exp\left(-\frac{k^2\mathcal{M}^2}{2}\right) \quad \text{(A4)}$$

and then taking the inverse Fourier transform of the previous expression. It leads to

$$p(m_{rj}) = \frac{1}{\sqrt{2\pi}\mathcal{M}} \exp\left(-\frac{m_{rj}^2}{2\mathcal{M}^2}\right). \quad \text{(A5)}$$

In that way we have proved that the random potential (8),

$$V_i = \frac{\phi_0}{4\pi\lambda} \sum_{j \neq i} \frac{\mathbf{m}_j \cdot \boldsymbol{\rho}_{ij}}{\rho_{ij}^2} = \frac{\phi_0}{4\pi\lambda} \sum_{j \neq i} \frac{m_{rj}}{\rho_{ij}} \quad \text{(A6)}$$

matches the random potential (16). We conclude that frozen magnetic dipoles parallel to the film create the same random potential for vortices in the film as magnetic dipoles normal to the film, provided both are Gaussian distributed.

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